Position predictive measurement method for time grating CNC rotary table

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Abstract

Time grating sensor transforms space domain information to time domain information and measures spatial displacement with time. To develop high precision time grating CNC rotary table and reduce the dynamic position feedback error of the table, circular position predictive measurement method is proposed for transforming time domain information back to space domain information based on time-space transformation technology. Predicted values are obtained by modeling the measured values with time series theory, and the last prediction error is corrected in real time using the next measured values. Modeling method and parameter estimation algorithm are presented. To conform the validity of the position prediction method, an experimental system is designed. The angle displacement prediction error of the rotary table is $\pm 2''$, and precise predicting is achieved.

Keywords: Predictive measurement, time-space transformation, time grating, CNC rotary table

1. Introduction

Time grating is a novel displacement sensor developed by the concept of measuring space with time, and angular displacement is measured very accurately without the use of a conventional graduated mechanical scale, which permits the greatly lowered cost of manufacture [1]. Time grating transforms space domain information to time domain information based on time-space transformation technology and measures spatial position every uniform time. In full closed-loop control mode CNC system requires position feeding every uniform space.

To use time grating as feedback component for CNC positioning servo control and develop high precision time grating CNC rotary table, the new theories and methods should to be developed for transforming time domain information back to space domain information based on time-space transformation technology. Prediction is currently used for industrial control successfully, predictive control has become a typical process control method, and predictive control theory and algorithm are widely studied [2]. Prediction conception was firstly introduced to measurement field [3], and this paper will develop circular position predictive measurement method to reduce the dynamic position feedback error of time grating CNC rotary table.

2. Principle of predictive measurement

According to the measurement principle of time grating [1, 4], rotating magnetic field is used to construct a moving coordinate system S' with constant velocity V, the spatial angular displacement θ between a moving probe P_a at random speed v and a fixed probe P_b can be measured by detecting the time difference ΔT that the rotating magnetic field scans the two probes (Fig. 1). I.e. $\theta = V\Delta T = (360^\circ)/(360^$ thus time grating gets a measured angle value every scanning period T i.e. measuring period.



Fig. 1. Time grating measurement model.

An absolute angle value of CNC rotary table is measured by time grating every time interval *T*. The *N* measured angle values θ_{i-N+1} , θ_{i-N+2} , ..., θ_i from time point T_{i-N+1} to T_i can be treated as a time series. Time series model is based on the correlation of the data, which describes the dynamic characteristic and variation rule of the series. Thus the future values of the time series can be predicted by using time series model [5]. The principle of predictive measurement method is shown in Fig. 2. At current time point T_i , the angle displacement of



Fig. 2. Principle of predictive measurement method.

the rotary table $\Delta \theta_i$ during the next measuring period *T* (from time point T_i to T_{i+1}) can be predicted by modeling the *N* absolute measured angle values θ_{i-N+1} , θ_{i-N+2} , ..., θ_i , then incremental pulses are outputted representing the angle displacement $\Delta \theta_i$ during the next measuring period. In this way, discrete absolute measured angle values can be transformed to continuous incremental pulses that are fed to CNC system. The angle displacement of the rotary table $\Delta \theta_k$ during the *k*th measuring period (from time point T_{k-1} to T_k) is:

$$\Delta \theta_k = \theta_k - \theta_{k-1}. \tag{1}$$

Using time series theory the predicted angle displacement value of the rotary table $\Delta \theta_{i+1}$ during the next measuring period (from time point T_i to T_{i+1}) can be obtained as:

$$\Delta \hat{\theta}_{i+1} = L(\Delta \theta_{i+1} | \Delta \theta_i, \Delta \theta_{i-1}, \cdots, \Delta \theta_{i-N+1}).$$
⁽²⁾

The number of pulse outputted in PWM (Pulse-Width Modulation) mode during the next measuring period (from time point T_i to T_{i+1}) is:

$$P_{i+1} = (\Delta \hat{\theta}_{i+1} - e_i) / Q,$$
(3)

where Q is the pulse equivalent and e_i is the prediction error of the last measuring period (from time point T_{i-1} to T_i). Time grating will get a new measured angle value θ_{i+1} when time point T_{i+1} come. θ_{i+1} can be used to calculate the prediction error between the predicted value and the measured value. Eq. (3) shows that the prediction error of the last measuring period is corrected when current predicting is conducted. And the prediction error will not be cumulated, which assures the aforementioned method of high precision.

3. Algorithm for position prediction

Prediction model is based on stationary data series. Firstly it is necessary to preprocess the dynamic data measured by time grating, remove the deterministic trends and transform non-stationary data series to stationarity [5, 6]. And we apply stationary data series to modeling, analysis and prediction for the residual data then finish modeling and analysis for current and past angle measured values of time grating together with the deterministic trends. Finally predicting angle displacement of the CNC rotary table is achieved.

The expression of *p*th order autoregressive model AR(p) for time series $\{X_t\}$ can be described as:

$$X_{t} = \sum_{j=1}^{p} a_{j} X_{t-j} + \varepsilon_{t}, t \in \Box, \qquad (4)$$

where $\{\varepsilon_t\}$ is white noise WN(0, σ^2), real number $a = (a_1, a_2, ..., a_p)^T$ is the autoregressive coefficient and $a_p \neq 0$. X_{n+1} can be successively predicted with p data $X_n, X_{n-1}, ..., X_{n-p+1}$. The expression for optimal linear prediction is:

$$\hat{X}_{n+1} = L(X_{n+1} | X_n, X_{n-1}, \cdots, X_{n-p+1}) = \sum_{j=1}^p a_j X_{n-j+1}.$$
(5)

Parameter estimation algorithm is as follows for autoregressive coefficient a. Firstly we need to preprocess the observed data $X_1, X_2, ..., X_N$ to obtain a new time series $\{Y_t\}$ with zero mean value:

$$Y_{t} = X_{t} - \overline{X}_{N}, t = 1, 2, \cdots, N$$

$$\overline{X}_{N} = \frac{1}{N} \sum_{j=1}^{N} X_{j}$$
(6)

Then we model the $\{Y_t\}$ with AR(*p*). The estimation for autocovariance function of the preprocessed data is:

$$\hat{\gamma}_{k} = \frac{1}{N} \sum_{j=1}^{N-k} y_{j} y_{j+k}, k = 0, 1, \cdots, p.$$
(7)

The square estimation for autoregressive coefficient and white noise $(\hat{a}_1, \hat{a}_2, \dots, \hat{a}_p)^T, \sigma^2$ are determined by Yule-Walker equation

$$\begin{bmatrix} \hat{\gamma}_{1} \\ \hat{\gamma}_{2} \\ \vdots \\ \hat{\gamma}_{p} \end{bmatrix} = \begin{bmatrix} \hat{\gamma}_{0} & \hat{\gamma}_{1} & \cdots & \hat{\gamma}_{p-1} \\ \hat{\gamma}_{1} & \hat{\gamma}_{0} & \cdots & \hat{\gamma}_{p-2} \\ \vdots & \vdots & & \vdots \\ \hat{\gamma}_{p-1} & \hat{\gamma}_{p-2} & \cdots & \hat{\gamma}_{0} \end{bmatrix} \begin{bmatrix} \hat{a}_{1} \\ \hat{a}_{2} \\ \vdots \\ \hat{a}_{p} \end{bmatrix}$$
(8)

$$\boldsymbol{\sigma}^{2} = \hat{\boldsymbol{\gamma}}_{0} - (\hat{a}_{1}\hat{\boldsymbol{\gamma}}_{1} + \hat{a}_{2}\hat{\boldsymbol{\gamma}}_{2} + \dots + \hat{a}_{p}\hat{\boldsymbol{\gamma}}_{p}).$$

$$\tag{9}$$

4. Experiment setup and results

Experiment setup is shown in Fig. 3. An AC servomotor drives a rotary table installed with a $\pm 0.8''$ time grating sensor acting as position feedback component. Siemens 802D digital CNC servo system works with SMC30 encoder interface module for full closed-loop position control. The CNC system receives the feedback signals from time grating and then



Fig. 3. Experiment setup.

precisely controls the position of the rotary table. HEIDENHAIN angular encoder ROD880 with 36000 line count and accuracy of $\pm 1''$ is mounted for testing the dynamic prediction error and the positioning accuracy of the rotary table. Interpolation and digitizing electronics IBV660B is applied for converting the ROD880 sinusoidal signals to TTL square-wave pulse trains as well as 400-fold subdivision. Consequently the measuring step of the testing system



Fig. 4. Tested position curve of rotary table.



Fig. 6. Acceleration curve of rotary table.



Fig. 5. Velocity curve of rotary table.



Fig. 7. Prediction results of negative acceleration motion.

and

is 0.09". A counter card is developed to process the outputted signals from IBV660B and the incremental pulses from time grating and can simultaneously lock the two counting results for synchronous displacement comparison. The card is connected to a computer via a RS-232C port, and the computer can automatically acquire the two locked data. The computer can also get the measured angle value of time grating directly with another RS-232C port.

Figure 4 shows a tested position curve of rotary table. The CNC system drove the table rotating from 41.98105° to 61.98172° within 7 seconds. The computer sampled the position of the rotary table ten times per second. There were four different motion status including rest, positive acceleration motion, approximate uniform motion and negative acceleration motion (Fig. 5). The angular acceleration curve of rotary table is shown in Fig. 6. Figure 7 illustrates the prediction results of negative acceleration motion (the fourth motion course denoted in Fig. 5) using AR(3) model. The acceleration prediction error varies form -0.00023"/ms² to 0.00019"/ms². And the prediction error goes bigger along with the larger acceleration variation. According to the relationship between displacement and acceleration, the angle displacement prediction error of the rotary table is ± 2 ".

5. Conclusion

This paper briefly introduced the principle of time grating. Circular position predictive measurement method was proposed to reduce the dynamic position feedback error of time grating CNC rotary table. Predicted values were calculated by modeling the measured values with autoregressive model, and the prediction coefficient computing algorithm was presented. The last prediction error was corrected in real time using the next measured values. Discrete absolute measured angle values were transformed to continuous incremental pulses that were fed to CNC system. Experimental results have shown that precise position predicting for time grating rotary table was achieved. Further research will focus on continuous predicting for composite motion.

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